the findings of the prior paragraphs; and so on for any number of films.

CONCLUDING REMARKS

Typical experimental reflectance data, taken from [2], are presented in Fig. 2. The data are for perpendicular polarized radiation at a wavelength of 0·633 μ. The upper and lower sets of data correspond respectively to an aluminum oxide film ($h = 1{\cdot}71\ \mu$) on an aluminum substrate and to a zirconium oxide film ($h = 1{\cdot}51\ \mu$) on an aluminum substrate. The solid lines represent the predicted reflectance versus angle distribution as evaluated from equation (4). Examination of the figure reveals that accurate reflectance information at $\theta_i = 0°$ can be obtained by using the properties derived here as a guide for extrapolating the data.

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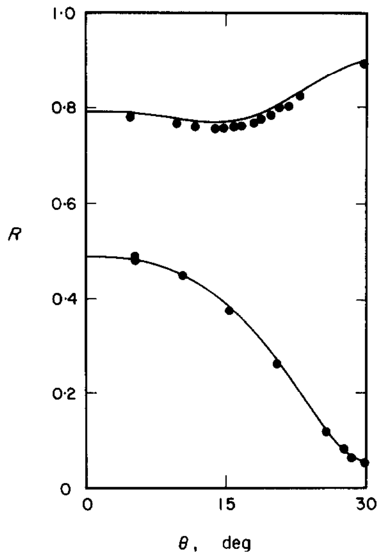

Fig. 2. Typical reflectance data

Int. J. Heat Mass Transfer. Vol. 16, pp. 1645–1648. Pergamon Press 1973. Printed in Great Britain

HEAT TRANSFER TO STEAM FLOWING TURBULENTLY IN A PIPE

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(*Received* 30 *August* 1972 *and in revised form* 8 *January* 1973)

NOMENCLATURE

a,	constant equal to 1·0;
A,	total band absorptance;
b,	constant equal to 1·25;
B,	radiation intensity;
c_p,	specific heat at constant pressure;
k,	thermal conductivity;
Nu,	$= q_0 2r_0/k(T_0 - T_b)$, Nusselt number;
q,	heat flux;
r,	radial coordinate;
Re,	$= u_b 2r_0/\nu$ Reynolds number;
T,	temperature;
u,	velocity.

Greek symbols

β,	$= q_0(\tau_0/\rho_0)^{\frac{1}{2}}/c_{po}\tau_o T_o$, heat-transfer parameter;
γ,	angle;
μ,	dynamic viscosity;
ν,	$= \mu/\rho$ kinematic viscosity;
ρ,	density;
τ,	shear stress;
ω,	wave number.

Subscripts

b,	bulk value;
c,	value at band center;
o,	evaluated at wall.

THE DETERMINATION of the energy transport in a radiating gas which is flowing in a circular tube is a difficult problem [1-6, 16]. The complexity of the calculations have resulted in many studies which have omitted the contribution from thermal radiation. The present study clearly demonstrates the importance of radiative transport for steam flowing turbulently in a 2 in. tube at the conditions given in Table 1 and Figs. 1-3. The results also show that the radiative transport for steam may be accounted for by the use of the total band absorptance.

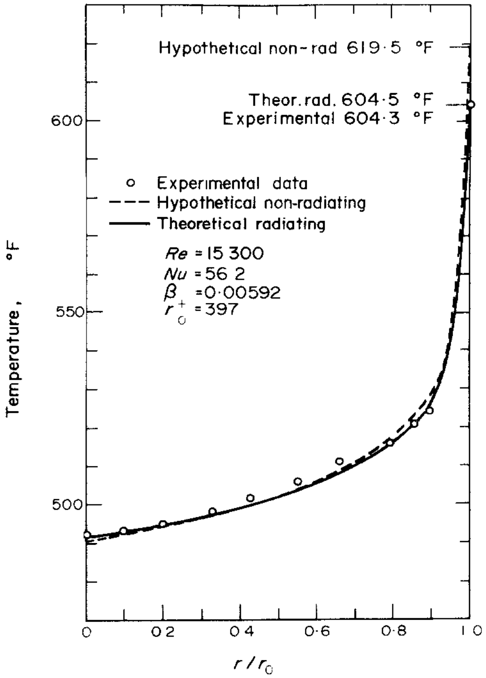

FIG. 1. Experimental and theoretical temperature profiles for steam.

Table 1. Summary of results

$Re_{0\cdot4}^*$	β	P(atm)	$Nu_{0\cdot4}^*$ (exp't., rad.)	Nu (hypothetical, non-rad.)
22100	0·00652	1·0	71·4	56·4
15300	0·00592	1·0	56·2	44·1
15300	0·00608	1·0	54·8	43·9

* Properties evaluated at $T_{0\cdot4} = T_b + 0\cdot4(T_0 - T_b)$ after Deissler and Eian (15).

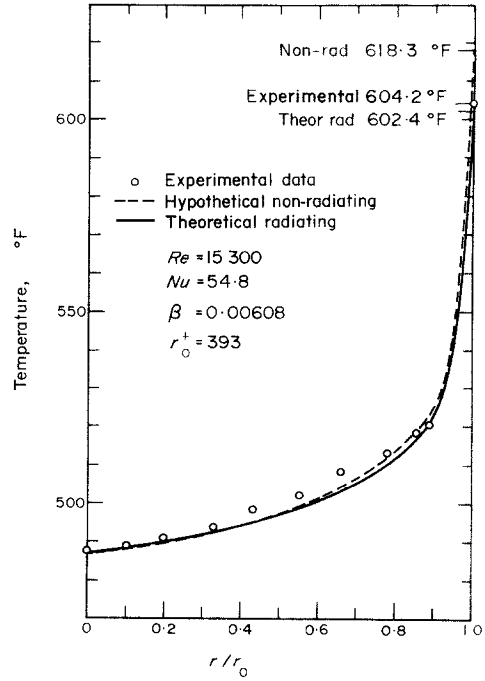

FIG. 2. Experimental and theoretical temperature profiles for steam.

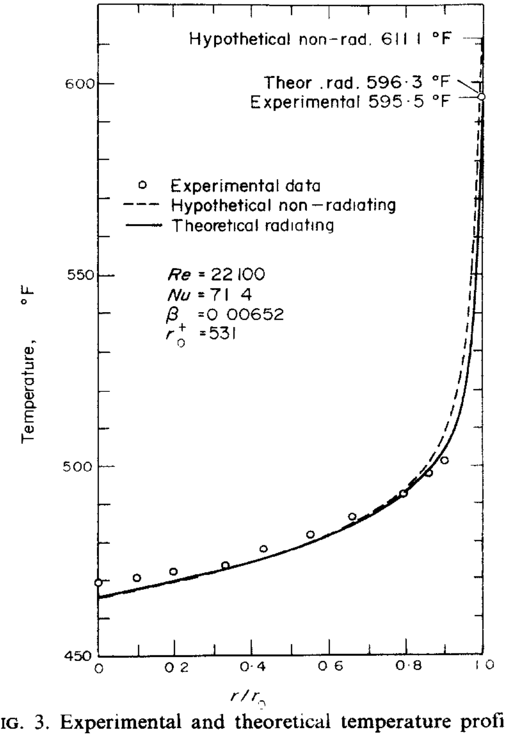

FIG. 3. Experimental and theoretical temperature profiles for steam.

In a previous study the heat transfer was determined in fully developed turbulent flow in a circular tube with a radiating gas, carbon dioxide, and with a non-radiating gas, air [5]. The present note is concerned with the turbulent flow of steam. The experimental apparatus is essentially described in reference [5] and will not be repeated here. The system was modified to accommodate steam and a detailed description is available [7].

The experimental temperature profiles for steam in a 2-in. electrically heated pipe at an axial location 108 tube diameters from entry are presented in Figs. 1–3. An informative comparison can be readily made between the experimental data, which obviously includes the radiation contribution, and a hypothetical non-radiating calculation which simply omits radiation but corresponds to the same wall flux, q_o, and mass flow rate $\dot{m}$. Thus, the bulk temperature is the same for both conditions. The effect of radiation is to increase the energy transport thereby decreasing the wall temperature and resulting in a flatter temperature profile. These results may be seen in the figures by comparing the experimental data points with the dotted hypothetical non-radiating curve. The major effect is on the wall temperature.

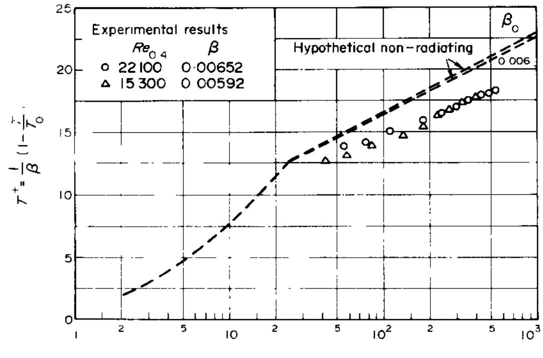

FIG. 4. Dimensionless temperature profiles for steam.

For completeness, the results are also presented on a dimensionless basis in Fig. 4. The effect of radiation may also be noted by comparing the experimental value of the Nusselt number with that obtained from the non-radiating calculations.* This comparison is made in Table 1 and a radiation contribution of approximately 20 per cent results.

The remaining consideration is to solve the energy equation including the effects of conduction, convection and radiation in a non-gray cylindrical medium. The radiative flux calculation has been made by introducing the total band absorptance [8, 9] into equation (24) of Kesten [10] and the resulting expression for the flux is given by

$$q_{\text{rad}}(r) = \frac{4a}{b} \int_{\gamma=0}^{\pi/2} \left\{ \int_{r\sin\gamma}^{r_0} (A[b|r\cos\gamma - S(r', r, \gamma)|] - A[b\{r\cos\gamma + S(r', r, \gamma)\}]) \times \frac{\mathrm{d}}{\mathrm{d}r'}[B\omega_c(r') - B\omega_{c,o}]\,\mathrm{d}r' \right\} \times \cos\gamma\,\mathrm{d}\gamma \qquad (1)$$

where

$$S(r', r, \gamma) = (r'^2 - r^2 \sin^2\gamma)^{\frac{1}{2}}$$

and the remaining quantities are defined in the nomenclature.

The infrared spectrum of steam has three bands at 2·7μ, 6·3μ and 20μ and the radiative flux is obtained by summing the contributions from each band. The first two are vibration–rotation bands and the last is a pure rotation band. The properties of the bands have been taken from references [11–14].

The complete calculation for the total energy flux follows, very closely, the analysis of reference [5] and the resulting temperature profiles are presented as solid curves in the figures. Good agreement between the experimental and theoretical results is obtained.

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* In response to the referee's comment we emphasize that a comparison between theory and experiment for a non-radiating case; that is, for air, was previosuly made [5]. Additional runs have also been made [7]. The good agreement that results provides a check on the experimental system and on the non-radiating calculations.

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Int. J Heat Mass Transfer Vol. 16, pp 1648–1651 Pergamon Press 1973 Printed in Great Britain

NUMERICAL SOLUTIONS OF THE INTEGRO-DIFFERENTIAL EQUATIONS OF HIGH-SPEED RADIATING BOUNDARY LAYERS

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(*Received* 19 *April* 1971 *and in revised form* 30 *June* 1972)

NOMENCLATURE

C_p,	specific heat at constant pressure;
E_n,	exponential integral function defined in equation (7);
e_b,	Planck's radiation function;
H,	stagnation enthalphy;
l,	mixing length;
Pr_{eff},	effective Prandtl number;
q_R,	radiant heat flux vector;
Re_x,	local Reynolds number;
T,	temperature;
u,	velocity, x-direction;
v,	velocity, y-direction;
x,	length, parallel to body surface;
y,	length, normal to body surface;
δ,	boundary layer thickness;
κ,	absorption coefficient;
μ,	laminar viscosity;
μ_{eff},	effective viscosity;
ρ,	density;
τ,	shear stress;
τ_λ,	optical coordinate defined in equation (5);
$\tau_{0\lambda}$,	optical thickness of boundary layer defined in equation (6);
Φ,	generation term for ϕ [5].

Subscripts

D,	downstream point in the finite difference grid;
R,	radiation term;
U,	upstream point in the finite difference grid;
w,	body surface condition;
λ,	wavelength;
0.	reference condition;
∞,	freestream condition.

Bold symbols indicate a vector quantity.

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